Journal of Business Finance & Accounting, 11(2), Summer 1984, 0306 686X $2.50

THE VALIDITY OF CONVENTIONAL VALUATION MODELS UNDER MULTIPERIOD UNCERTAINTY

ØYVIND BØHREN*

Ever since the introduction of the time-state-preference (TSP) model, a broadly accepted valuation principle for multiperiod cashflows has been available (Debreu, 1959, ch. 7). However, this introduction of what might be termed a complete model (Demski, 1980) did not resolve the real-world difficulties. Operationalizing the TSP approach has so far been prohibitively costly, although significant progress has recently been made via option pricing (Breeden and Litzenberger, 1978, and Banz and Miller, 1978). Thus, several conventional models which are clearly simplified relative to the TSP continue to be used in practice (Schall et al., 1978). Such models are the simulation procedure of Hertz (1964 and 1968) and the related mean/variance model of Hillier (1963 and 1968), the certainty equivalent (CE) approach (Robichek and Myers, 1965 and 1966), the risk adjusted discount rate (RADR) procedure (Robichek and Myers, 1965 and 1966), and also some version of the internal rate of return (IRR) model (Hertz, 1964 and 1968; Lindsay and Sametz, 1963, ch. 3; and Fairley and Jacoby, 1975). Thus, the decisionmaker may choose from at least four groups of simplified models to solve his cashflow valuation problem.

This widespread use of simplified models in competitive environments suggests that although decision errors do occur relative to a more sophisticated approach, lower costs of model construction and data estimation may still favour conventional approaches in net benefit terms. While it is hard to evaluate the latter costs without assuming a quite particular decision context, case-independent conclusions on potential *decision errors* are easier to derive. By assessing previous contributions and by developing new insights, this paper uses an analytical approach to evaluate relative merits of four conventional models from a decision error point of view. The TSP model serves as a standard of comparison.

The conventional models are briefly presented in the first section, followed by an appraisal of their original choice-theoretic background. The second section explores the models' consistency with TSP valuation under various

* The author is Associate Professor of Finance at the Norwegian School of Management. He wishes to acknowledge the valuable comments on previous versions of this paper by Professors Steinar Ekern, Lars Mathiesen and Gary L. Sundem. (Paper received April 1983, revised September 1983)

assumptions about data observability, cashflow type, investor beliefs, and market valuation parameters. The final section summarizes the findings.

THE CONVENTIONAL MODELS

Definitions Hertz and Hillier (HH) proposed the criterion function

$$\mathrm{VHH} = \lambda \sum_{t=0}^{T} \bar{X}_t (1 + i_t)^{-t} = \mathrm{NPV}(\bar{X}, i).$$

VHH is cashflow value in the HH model, $\bar{X} = [\bar{X}_t]$ is the expectation of the stochastic cashflow $\tilde{X} = [\tilde{X}_t]$, $i = [i_t]$ is the set of riskless rates,[1] and λ is a risk adjustment factor. Thus, cashflow value is found by first pricing cashflow $\tilde{X}$ as though it paid off its expectation $\bar{X}$ with certainty, subsequently considering risk through λ. Supposedly, λ reflects cashflow risk as well as a unit cost of risk.

In Hillier's version, risk is the variance of $\mathrm{NPV}(\tilde{X}, i)$, which is derived analytically from all the separate variances of the components $\tilde{X}_t$. Hertz used Monte Carlo simulation to approximate the full $\mathrm{NPV}(\tilde{X}, i)$ distribution, and λ may thus reflect any of its distributional parameters. Obviously, that may also be accomplished by Hillier's model, provided $\mathrm{NPV}(\tilde{X}, i)$ is normally distributed.[2]

In the *certainty equivalent* (CE) approach,

$$\mathrm{VCE} = \sum_{t=0}^{T} \hat{X}_t (1 + i_t)^{-t} = \sum_{t=0}^{T} \alpha_t \bar{X}_t (1 + i_t)^{-t},$$

where the CE-factor α_t of $\tilde{X}_t$ transforms its expectation $\bar{X}_t$ into a certainty equivalent $\hat{X}_t$.

In the CE model, time t uncertainty is adjusted to time t certainty and then carried back to time zero at the riskless rate. Thus, whereas the CE model allows for a separate risk adjustment period by period, the HH procedure only has one adjustment factor over the entire cashflow life.

In the *risk adjusted discount rate* (RADR) approach, value is defined as

$$\mathrm{VRADR} = \sum_{t=0}^{T} \bar{X}_t (1 + r_t)^{-t},$$

r_t denoting the RADR converting $\bar{X}_t$ directly into a present value. Clearly, $(1 + r_t)^{-t}$ of VRADR corresponds to $\alpha_t(1 + i_t)^{-t}$ in VCE. They are both time zero prices of a unit expectation of $\tilde{X}_t$.

Finally, the *internal rate of return* (IRR) is usually defined in terms of expected cashflow $\bar{X}$. Letting $k_{\bar{X}}$ be the discount rate making VRADR equal to zero, the implicit criterion definition is

$$0 = \sum_{t=0}^{T} \bar{X}_t (1 + k_{\tilde{x}})^{-t}.$$

Although this model assigns no value to $\tilde{X}$, it may still be used for decision-making. $\tilde{X}$ is accepted if its criterion value $k_{\tilde{x}}$ exceeds the hurdle rate $h_{\tilde{x}}$.[3]

Choice—theoretic background The justification originally given for *the HH model* seems to rely on basic results from two widely different decision contexts. First, under multiperiod certainty, NPV at the riskless rate is consistent with standard preference assumptions. Second, given timeless uncertainty, the mean/variance rule has a corresponding property under normal probability distributions. Thus, it is argued, with both time and uncertainty, mean and variance of NPV should be used, both discounted at riskless rates (Hillier, 1963, pp. 445–47 and 1969, pp. 2–5, and Van Horne, 1966 and 1969).

Such an argument is quite arbitrary. Clearly, there is no *a priori* reason why the valuation principle for multiperiod, stochastic cashflows should be just the "sum" of the two extreme cases of multiperiod certainty and timeless uncertainty.

Like HH, *the CE approach* merges the certainty-based NPV model with ideas from the economics of uncertainty literature. Specifically, if the (timeless) preference function for the uncertain $\tilde{Z}$ is $U(\tilde{Z})$, the certainty equivalent $\hat{Z}$ is implicitly defined by $U(\hat{Z}) = EU(\tilde{Z})$, where E is the expectation operator. Going from here to multiperiod uncertainty of cashflow $\tilde{X}$, keeping the NPV model in mind, the reasoning seems to be that once the uncertainty of $\tilde{X}_t$ is accounted for by $\hat{X}_t = \alpha_t \bar{X}_t$, the only remaining adjustment is for time difference between 0 and t (Lutz and Lutz, 1951, ch. 15, and Robichek and Myers, 1965, pp. 79–81 and 1966, pp. 727–28).

Once more, this is ad-hoc. VCE was originally not derived from basic assumptions about the decision context, such as cashflow properties, investor preferences, and market opportunities. It therefore leaves unanswered issues like why α_t is just a function of $\tilde{X}_t$ and not of the entire $\tilde{X}$, of total investor wealth or even of aggregate market wealth.

Even *the RADR* originally built on the NPV criterion from multiperiod uncertainty. Subsequently observing that X_t is risky, i_t is raised by a risk correction factor, implicitly assuming that a certain amount is worth more than a stochastic one with the same expectation (Lutz and Lutz, 1951, ch. 15 and Robichek and Myers, 1965, p. 80).[4]

Finally, *the IRR approach* starts with the corresponding certainty model, substituting the deterministic cashflow by the expectation $\bar{X}$ of $\tilde{X}$ (Lindsay and Sametz, 1963, pp. 44–54). Moreover, the hurdle rate is raised above i_t to account for risk as well as timing. Alternative versions generate the expected IRR or its complete or approximate distribution (Hertz, 1964 and 1968; Hillier, 1963; and Fairley and Jacoby, 1975).

Consequently, the four conventional models were not developed from coherent assumptions about cashflows, investors or market opportunities. They built on valid results under certainty, but were transformed to multiperiod uncertainty in an ad-hoc way. They all tend to focus on individual utility functions and isolated cashflows rather than market valuation processes. Only the HH model defines its risk measure, and none of them prices risk. Thus, risk adjustment is in fact *exogeneous* input in all four models. Some external valuation principle is required before cashflow value can be computed.

Having reached this conclusion, it is essential to notice that some of these models have been properly justified and fully specified *ex post*, i.e., after having been introduced. First, the single-period CAPM may be stated as a CE or a RADR version. Moreover, the CAPM is easily shown to be a special case of TSP (see for example Bøhren and Ekern, 1981). Second, for multiperiod uncertainty, Fama (1977) developed a RADR model assuming the single-period CAPM to hold period by period and that future riskless rates, prices of risk, and cashflow covariances are deterministic. Still, however, both results are insufficient in the present context, as the results apply to only one particular environment (the CAPM world) and just a subset of the simplified models (CE and RADR).

The most general result is due to Rubinstein (1976). He proved the existence of a CE model by assuming just state-contingent payoffs, the single-price law, and non-satiation. To operationalize the model, however, a TSP environment and alternative sets of more restrictive cashflow and preference properties were required. Using one particular context, Leland (1980) subsequently developed a RADR model. The significance of these results will be considered in the next section under the heading Operational consistency.

There have been previous attempts to relate some of the conventional models directly to TSP, and this work is evaluated in the next section under the heading Consistent valuation of $\tilde{X}_t$.

TSP CONSISTENCY OF CONVENTIONAL MODELS

In the TSP model, the value of $\tilde{X}$ is

$$V = V(\tilde{X}) = \sum_{t=0}^{T} V_t = \sum_{t=0}^{T} \sum_{s=1}^{S(t)} X_{st}\phi_{st}.$$

V and V_t are the present (time zero) values of $\tilde{X}$ and $\tilde{X}_t$, respectively. $S(t)$ is the number of time t states, X_{st} is the payoff at t if s occurs, and ϕ_{st} is the present price of a claim to \$1 in state s at t.[5] The riskless rate is

$$\phi_t = \sum_s \phi_{st} = (1 + i_t)^{-t}.$$

Thus, ϕ_t is today's price of one deterministic time t dollar, and the riskless rate is $i_t = \phi_t^{-1/t} - 1$. If cashflows offered by firms span the time-state space or if investors can freely partition firm flows, every component of TSP valuation is a market parameter, independent of the specific investor's preferences or beliefs.[6]

As the TSP model is separable and additive over time, $\tilde{X}_t$ can be valued independently of the remaining components in $\tilde{X}$. Moreover, expected cashflow has no particular significance for valuation. What matters are contingent payoffs X_{st}, not individual beliefs P_{st} reflected in $\bar{X}_t = \sum_s X_{st} P_{st}$, where P_{st} is the subjective probability of state s occurring at t.

The four simplifications may now be compared to the complete TSP along two basic lines. First, disregarding all data problems, one may examine the decision errors of any simplified model m_j (HH, CE, RADR or IRR) relative to the complete m_c (TSP). This may be termed *theoretical consistency*, telling a model's potential in the ideal context where all parameters of m_c are known. The second issue is *operational consistency*, i.e. real-world usefulness of m_j versus m_c under imperfect knowledge about the parameters of m_c. Theoretical consistency will be analyzed first, followed by operational consistency.

One methodological qualification is required. From a decision error standpoint, the simplified model is consistent with the complete model (and thus fully valid) if *identical decisions* result; this will occur if the two models rank cashflows in the same way. What is important is the relative ordering of alternatives, not the value assigned to them per se. However, except for the IRR, it seems analytically infeasible to compare the present models in this way. Instead, *criterion functions* are compared, and it is argued rather heuristically that relative valuation differences tend to yield ranking discrepancies as well. Although it is never stated, all previous works in this area, as well as analyses of other models (Sundem, 1974) use this approach as a substitute for the ideal.

Consistent valuation of $\tilde{X}$ Panel (a) of Table 1 lists the criterion functions of the four simplifications, where $\phi_{r_t} \equiv (1 + r_t)^{-t}$ in RADR is the present price of one unit expectation of $\tilde{X}_t$. Panel (b) states consistency conditions if the only requirement is that the same *total value* is assigned to the entire T-period cashflow $\tilde{X}$ as does the TSP. In *HH*, the single risk adjustment factor λ must equal the ratio between time zero value of $\tilde{X}$ and its value if it always paid off its expectation $\bar{X}$. Thus, λ exceeds 1 if $\tilde{X}$ is more valuable than a cashflow promising $\bar{X}$ with certainty, and less than 1 otherwise.[7] If $\tilde{X}$ is deterministic ($X_{st} = X_t$ at every date), then $\lambda = 1$. Consequently, the HH model may potentially yield the TSP value of $\tilde{X}$.

It can be seen from panel (b) that λ is unique. However, the *CE and RADR* both involve one equation in $T + 1$ variables of α_t and ϕ_{r_t} respectively, producing an infinite number of theoretically consistent solutions. Consequently, many valid risk adjustments may yield the TSP value of $\tilde{X}$, and some of them should be explicitly discussed.[8] First, a constant adjustment $\alpha_t = \alpha$ in CE yields

Table 1

Requirements for TSP Consistency $V = \sum_t V_t = \sum_{s,t} X_{st}\phi_{st}$

		General case		Special cases, $\tilde{X}_t$ valuation		
Model	(a) Criterion	(b) $\tilde{X}$ valuation	(c) $\tilde{X}_t$ valuation	(d) $X_{st} = X_t$	(e) $\phi_{st} = \bar{\Phi}_t$	(f) $c_t = \phi_{st}/P_{st}$
HH	$\lambda \sum_t \bar{X}_t\phi_t$	$\lambda = V/\sum_t \bar{X}_t\phi_t$	$\lambda = V_t/\bar{X}_t\phi_t$	$\lambda = 1$	$\lambda = \check{X}_t/\bar{X}_t$	$\lambda = 1$
CE	$\sum_t \alpha_t\bar{X}_t\phi_t$	$\sum_t \alpha_t\bar{X}_t\phi_t = V$	$\alpha_t = V_t/\bar{X}_t\phi_t$	$\alpha_t = 1$	$\alpha_t = \check{X}_t/\bar{X}_t$	$\alpha_t = 1$
RADR	$\sum_t \bar{X}_t\phi_{r_t}$	$\sum_t \bar{X}_t\phi_{r_t} = V$	$\phi_{r_t} = V_t/\bar{X}_t$	$\phi_{r_t} = \phi_t$	$\phi_{r_t} = \phi_t X_t/\bar{X}_t$	$\phi_{r_t} = \phi_t$
IRR	$k_{\tilde{x}}$	$h_{\tilde{x}} = (\bar{X}_t/V_t)^{1/t} - 1$	—	—	—	—

the HH model. Second, a constant RADR $r_t = r$ is the solution to

$$\sum_t \bar{X}_t(1 + r)^{-t} - V = 0.$$

Clearly, T roots exist. Whether there is just one real, non-negative RADR depends on standard IRR uniqueness conditions (Pratt and Hammond, 1979). Third, the lacking economic sense of some cases is demonstrated by the CE solution of $\alpha_t\bar{X}_t = X = V\left[\sum_t \phi_t\right]^{-1}$, yielding X as the riskless annuity of V. No matter how $\tilde{X}$ is distributed over dates and states, the same certainty equivalent cashflow may be assigned to every date by setting $\alpha_t = V\left[\sum_t \phi_t\right]^{-1}/\bar{X}_t$.

Apart from the uniqueness problem of *the IRR*, consistency requires the ratio $\bar{X}_t/V_t$ to be a geometric series with a constant parameter.[9] Thus, unlike the others, the IRR model cannot be consistent with TSP unless a quite particular cashflow type is involved. If no such restrictions are made, a sufficient consistency condition is that the ratio of probabilities to state prices is constant across states and follows a geometric time series. However, as will be noted later in this section under the heading Operational consistency, probabilities proportional to prices implies a riskless hurdle rate, such that no risk adjustment is called for whatsoever.

Any of these assumptions about cashflows, personal beliefs and prices is strong, suggesting that decision errors may be frequent. Hence, the IRR model is clearly inferior to HH, CE and RADR, which can handle any pattern of cashflows, probability beliefs, and price structures.

As a final point, there is no case where risk adjustment depends on the preferences of the individual evaluator. However, *personal beliefs* do enter the simplified models through the subjective probability of state occurrence. As these may vary across individuals, λ, α_t, r_t, $k_{\tilde{x}}$ and $h_{\tilde{x}}$ are not market parameters in a TSP environment. In contrast, the homogeneity assumption of the CAPM precludes investor-specific valuation parameters.

Consistent valuation of $\tilde{X}_t$ The lack of economic sense was demonstrated by the annuity case of $\alpha_t = V\left[\sum_t \phi_t\right]^{-1}/\bar{X}_t$. Thus, although all conventional models may potentially be correct, the preceding analysis does not tell the whole story about their relative merits. Moreover, the true value V of a potential cashflow $\tilde{X}$ is normally unknown at the time of analysis. Therefore, the only practical approach is to construct the parameters of the simplified models without knowing V. It would then happen only by chance that undervaluations of some $\tilde{X}_t$ are exactly washed out by corresponding overvaluations. Consequently, unless a simplified model may potentially valuate $\tilde{X}$ correctly *period by period* through V_t, it will not produce the correct total value V.

Panels (b) and (c) show that correct period by period valuation is sufficient, but not necessary, for consistent valuation of the entire $\tilde{X}$. Any other solution always involves an incorrect value of one or more $\tilde{X}_t$. This is not surprising, as panel (b) only requires total undervaluations to equal total overvaluations.

For *the HH model* to yield V_t, the ratio of time t certainty equivalent (V_t/ϕ_t) to time t expected cashflow $(\bar{X}_t)$ must be constant at each point in time. Whereas periodic risk adjustments in CE and RADR can be based on $\tilde{X}_t$ alone, HH therefore makes assumptions about the entire $\tilde{X}$. Although the separability of TSP justifies a period by period approach, this advantage cannot be utilized by HH.

The RADR, unlike *the CE*, cannot correctly evaluate the time zero cashflow component unless its true value equals $\bar{X}_0$ $(\bar{X}_0\ (1 + r_0)^{-0} = \bar{X}_0$ for any $r_0)$. Moreover, constant risk adjustment parameters means $\alpha_t = \alpha$ in CE and $r_t = r$ in RADR. The former is TSP consistent only when HH is TSP consistent. A constant RADR is only consistent period by period when the IRR causes no decision errors valuing $\tilde{X}$. Clearly, the two requirements are qualitatively different, and it is not obvious which is the more restrictive (Bøhren, 1983). Finally, besides what was stated in respect of *the IRR model* under the heading Consistent valuation of $\tilde{X}$, there is nothing to add. Consistent $\tilde{X}_t$ and $\tilde{X}$ valuation are indistinguishable in that model.

The four conventional models have one *common feature* not shared by the TSP. The latter incorporates state-contingent cash and probabilities over states. The simplifications, however, have one dimension less, defining probabilities directly over cash. Hence, they cannot explicitly account for the possibility that the present value of one dollar at any date depends on the state in which it is to be received. To see that this can only be indirectly accounted for, consider the condition for a consistent $\tilde{X}$ value in RADR:

$$\sum_{s,\,t} \phi_{r_t} P_{st} X_{st} = \sum_{s,\,t} \phi_{st} X_{st}. \tag{1}$$

Clearly, the product $\phi_{r_t} P_{st}$ in RADR plays the role of the state price ϕ_{st} in TSP.

The significance of this is discussed below under the heading Operational consistency.

Some of the theoretical consistency issues analyzed here have been discussed elsewhere. Keeley and Westerfield (KW) (1972) claim that HH is inconsistent with TSP for valuing $\tilde{X}$ (p.704, note 5; p. 705, exp. 4; and p.706, exp. 13). That assertion is incorrect, as shown by row 1 of panel (b) in Table 1. KW derive no analytical expression for a consistent λ. Using one example, they miss the point that cashflows with different $V/\sum_t \phi_t \bar{X}_t$ ratios also have different λ values under consistent HH valuation. Moreover, their claim that CE and RADR are superior to HH is unsupported by their analysis, as no consistency requirement is developed for any of them. In fact, panel (b) refutes their assertion, since all three models may potentially yield V. What does hold, but is not addressed by KW, is the conclusion of panel (c) that the HH is less general than the CE and RADR for period by period valuation.[10] However, that is a different and stronger requirement than just consistent $\tilde{X}$ valuation.

Myers (1968, p.20) develops a mathematical relationship between $\hat{X}_t$, α_t and $\bar{X}_t$ in terms of TSP values, but no further analysis is made. Hirshleifer (1970, p.250) discusses the RADR in a two states-one period context with uncertainty only at the end of the period, such that $\tilde{X}_t$ versus $\tilde{X}$ valuation is indistinguishable. Haley and Schall (1979, pp.227–28) consider multiperiod RADR valuation of $\tilde{X}_t$, concluding that if appropriate RADRs are chosen, the model is TSP consistent. Like Hirshleifer, they note that RADRs are market parameters only under homogeneous expectations, and that if two cashflows differ by a scale factor across the $S(t)$ states, they have the same r_t.[11]

Going from consistent valuation of $\tilde{X}$ to consistent $\tilde{X}_t$ valuation, it was noted that because V is unavailable *a priori*, the simplified models can only perform properly if they yield correct values period by period. That issue has now been addressed. From a *practical* point of view, however, estimating all the V_t is even worse than estimating just V. So, although theoretically consistent $\tilde{X}_t$ valuation is an important starting point, determining proper risk adjustments according to panel (c) is practically infeasible, as V_t is unknown. Moreover, if V_t really *were* known, subsequently constructing a simplified model which is known to yield identical decisions would not make much sense. Consequently, real-world model comparisons must be based on performance under readily observable parameters like cashflow properties, personal beliefs or qualitative market characteristics. This issue is considered next.

Operational consistency In particular environments, the simplified model m_j always yields the same decisions as the complete m_c, even though neither V nor V_t is known to the user of m_j. Several such contexts are now constructed, relating to either cashflows, state-contingent prices, personal beliefs, or joint properties of cashflows and preferences.

The time t cashflow of panel (d) in Table 1 is *deterministic*. According to every model, no risk adjustment should be made, as $V_t = X_t\phi_t$.

Whereas (d) relates to cashflows, column (e) involves a *state-independent price structure* $\phi_{st} = \bar{\phi}_t$, i.e., every time t state is equally valuable. Then, the state-averaged claim $\check{X}_t \equiv \sum_s X_{st}/S(t)$ is the time t value of $\tilde{X}_t$. Hence, the state distribution is irrelevant for valuation, as a claim to $S(t)X_t$ in one state and nothing otherwise is as valuable as a deterministic claim to X_t in each of the $S(t)$ different states.

In panel (f), *probabilities are proportional to prices* at t.[12] However, as shown by Bøhren and Ekern (1981), this is only compatible with degeneracy in either market risk aversion or in randomness of market return. Thus, although it seems peculiar that no risk adjustment applies whatever the properties of $\tilde{X}_t$, the explanation is simple in a narket context.

This result may be used to demonstrate a basic difference between the complete and the simplified models. Using the RADR as an example, equation (1) showed that the product $\phi_{rt}P_{st}$ in RADR corresponds to ϕ_{st} in TSP. Thus, a sufficient, although not necessary, condition for consistent $\tilde{X}_t$ valuation is $\phi_{rt}P_{st} = \phi_{st}$, i.e., probabilities proportional to prices. In other words, $\phi_{rt}P_{st}$ is an exact counterpart of ϕ_{st} for a non-deterministic $\tilde{X}_t$ only in case (f). If that holds, however, no risk adjustment should be made, despite the fact that $\tilde{X}_t$ is uncertain.

The weakness of the HH model is apparent in (d) and (f), where a correct valuation of $\tilde{X}_t$ requires $\lambda = 1$. However, in order for the *remaining* $\tilde{X}_r \in \tilde{X}(r \neq t)$ to be correctly valued, either all of them must be deterministic or probabilities must always be proportional to prices when $\tilde{X}_r$ is stochastic. In case (e), where prices are state-independent, the remaining $\tilde{X}_r \in \tilde{X}$ are correctly valued only if the ratio of time t value to expected cashflow equals the ratio of average to expected cashflow at all dates.

The only information needed in cases (d), (e), and (f) is the risk-free rate, state-contingent payoffs, and subjective probabilities. A slightly different approach is used by Rubinstein (1976) and Leland (1980). To derive consistent risk adjustments in CE and RADR respectively without knowing V or V_t, certain joint assumptions are made about *all* cashflows and investors. For instance, Leland's discount rates are imputable from cashflow characteristics alone if utility functions exhibit constantly proportional risk aversion and are additively separable over time. Moreover, the periodic rates of return in the market and on $\tilde{X}$ must both be intertemporally independent. Even then, however, the evaluator must know an aggregate property of all investors which is not readily observable except when all have the same relative risk aversion. Thus, as usual, generality is traded off for operationality.

One common way of simplifying the estimation problem is by *a priori bounding* unknown parameters. For instance, $0 \leqslant \alpha_t \leqslant 1$ or $r_t \geqslant i_t$ may be assumed before estimation starts. Or, parameters may be assumed to follow particular patterns over time, the constant case being the simplest and most common one. Clearly, such restrictions imply certain general assumptions about $\tilde{X}$ and the way it is valued. That issue has recently been analyzed in detail (Bøhren, 1983).

Consequently, several cases are relatively easy to operationalize. In all the remaining cases, however, it may be asked whether there is a reasonable chance of correctly adjusting for risk when V and V_t are unknown. In HH, for instance, risk is the variance of NPV($\tilde{X}$, i). Clearly, when different cashflows have different ratios of total risk to market risk, this model would easily produce decision errors unless λ were correspondingly adjusted across cashflows. In the three remaining models risk parameters are exogeneous, i.e., not even defined by the model. Consequently, *any* risk adjustment may be used for determining α_t, r_t and $h_{\tilde{X}}$ respectively in CE, RADR and IRR models. In such cases, the probability of decision errors critically depends on how closely the externally determined risk parameter resembles that implied by TSP.

This highlights a basic problem of any model comparison. Theoretical and operational consistency of simplification m_j depends on the metric m_c. In this paper, the TSP model was chosen as m_c because it is the most general and theoretically well-founded one. However, with a different standard of comparison, different relative merits would easily result. For instance, if legal restrictions only allowed individuals to hold single cashflows and no portfolios, using the TSP as m_c is unreasonable. If a normatively correct model were used as m_c in such a case, the total risk feature of HH would turn from a weakness into a strength. Or, if the decision maker, for some strange reason, always preferred the ranking of the IRR model, that would be a suitable m_c, and the TSP model would again be inferior.

SUMMARY

As the ranking of alternatives depends on the values assigned to them, a decision analysis also includes the choice of a valuation model. Although decision errors are less frequent the more complete the model, these benefits must be traded off against increasing modelling and data costs. This paper explores the decision errors of four conventional models, using the time-state-preference (TSP) model as a standard of comparison. Various sets of assumptions are made about data observability, cashflow properties, investor beliefs, and market valuation parameters.

The Hertz/Hillier (HH), certainty equivalent (CE), risk-adjusted discount rate (RADR) and internal rate of return (IRR) models all merged valid results from multiperiod certainty and timeless uncertainty in an ad-hoc fashion. Their justifications focused on individual preferences and separate cashflows rather than a coherent market valuation framework, and no model tells how to account for risk.

Provided the TSP value V of cashflow $\tilde{X} = [\tilde{X}_t]$ is known, the HH, RADR and CE models may all yield correct decisions. There is just one such risk adjustment factor in HH, but an infinite number of consistent, periodic CE-factors α_t and RADRs r_t exist, including the constant $\alpha_t = \alpha$ and $r_t = r$. In the IRR model, however, restrictive assumptions must be made about $\tilde{X}$ and the way it is valued.

For conventional models to be TSP consistent period by period, every $\tilde{X}_t \varepsilon \tilde{X}$ must be correctly valued. Under this stronger requirement, HH only performs properly if the ratio of time zero TSP value V_t to expected cashflow $\bar{X}_t$ is constant at every date. No such assumptions are required by CE and RADR.

Theoretical consistency tells a model's potential under ideal conditions, i.e., when V or V_t is known *a priori*. However, that is only a first, although important, criterion for model choice, as these values are unavailable in practice. Operational consistency explores the decision error issue in the more realistic context where such data are unknown to the decisionmaker.

If either $\tilde{X}_t$ is deterministic, prices are state-independent, or subjective probabilities are proportional to prices at t, the only required market parameter is the riskless rate. Along with $\tilde{X}_t$ and personal beliefs, consistent model parameters may be computed from that information alone. Also, under certain joint assumptions about investor preferences and the time series of all stochastic cashflows, proper risk adjustments rely on just riskless rates and some distributional properties of cashflows. Finally, although estimation may be simplified by *a priori* restricting risk parameters, particular assumptions are implicitly made about $\tilde{X}$ and its valuation.

The analysis demonstrates a common problem of any comparative model study. As conclusions about relative merits always depend on the standard of comparison, it is essential to pick this with care. In this paper, the TSP model was used because it is considered the most general and theoretically well-founded valuation principle. Other choices would easily produce different results.

NOTES

1 Conventionally, $i_0 \equiv 0$ throughout the paper, that is, the present price of a certain, present dollar is set equal to unity.

2 Although Hertz focuses on the IRR, he suggests using the procedure to generate NPV distributions as well (1968, p.105). He does not discuss discount rates, but others explicitly recommend the riskfree ones (Van Horne, 1966, p.84 and 1969, p.378; Hillier, 1963, p.447 and 1969, p.4; and Lewellen and Long, 1972, p.22).

3 Two points should be noted. First, although the IRR model may also handle mutually exclusive cashflows, only the accept/reject case is discussed, as that demonstrates the model's general properties under uncertainty. Second, the IRR may be defined in terms of $\bar{k}_{\tilde{x}} =$ the expected IRR as given by its probability distribution (Hertz, 1965 and 1968, and Fairley and Jacoby, 1975). Due to Jensen's inequality, $k_{\bar{x}} > \bar{k}_{\tilde{x}}$ as the IRR is concave in $\tilde{X}$. For analytical convenience, $k_{\bar{x}}$ is used here, as there are no principal differences between them relative to the TSP.

4 This does hold for any risk-averter who disregards any other market opportunity. It follows from concave utility functions in the economics of uncertainty framework.

5 ϕ_{st} is determined by the supply and demand of state-contingent claims, the latter depending on the wealth distribution, preferences and beliefs. Generally, ϕ_{st} differs across s and t and is higher the more money is valued in state s relative to other states and the closer t is to the present.

6 Of course, prices and values do depend on individual characteristics in the sense that ϕ_{st} is an equilibrium price. However, once the ϕ_{st} have been established, $\tilde{X}$ may be valued solely in terms of contingent payoffs X_{st} and market prices ϕ_{st}, disregarding the specific beliefs or preferences of the evaluator.

7 If V and $V_F \equiv \sum_t \bar{X}_t \phi_t$ are both negative or V is positive and V_F negative, the opposite is true.

8 If certainty prevails at $t = 0$, there is one equation in T variables, as $\alpha_0 = \phi_{r0} = 1$. Hence, for single period cashflows, unique solutions exist if $\tilde{X}_0$ is deterministic.

9 If $h_{\tilde{x}}$ is the hurdle rate, it must have the property that whenever $k_{\tilde{x}}$ exceeds, equals or is less than $h_{\tilde{x}}$, then V is respectively greater than, equal to or less than zero.

If $\bar{X}_t/V_t = (1 + g)^t$, then $V_t = \bar{X}_t(1 + g)^{-t}$, and $V = \sum_t \bar{X}_t(1 + g)^{-t}$.

Consequently, for the IRR to be TSP consistent, the hurdle rate $h_{\tilde{x}}$ must equal g.

As $V_t = \sum_s X_{st}\phi_{st} = \sum_s X_{st}P_{st}\,(1 + g)^{-t}$, a sufficient, but not necessary, condition is $P_{st}/\phi_{st} = (1 + g)^t$.

10 In footnote 5, p.704, KW contend that unlike CE and RADR models, HH cannot be derived as special cases of TSP. As shown by panel (c) in Table 1, that is incorrect.

11 The latter also holds for a given individual under heterogeneous expectations, as probabilities are defined over states rather than payoffs. However, that does not imply that this r_t is identical across investors.

12 If the only assumption is state-independent probabilities of $P_{st} = P_t = 1/S(t)$ (i.e., the uninformed case), no simplification occurs relative to panel (c).

REFERENCES

Banz, R.W. and M. Miller (1978), "Prices for state-contingent claims: Some estimates and applications", *Journal of Business* (1978, Vol. 51, No. 4), pp.653–672.

Breeden, D.T. and R.H. Litzenberger (1978), "Prices of state-contingent claims implicit in option prices", *Journal of Business* (1978, Vol.5, No.4), pp.621–651.

Bøhren, Ø. (1983), "Bounding certainty equivalent factors and risk-adjusted discount rates", *Journal of Business Finance and Accounting* (1983, Vol.10, No.1) pp.139–146.

——— and S. Ekern (1981), "Probabilities proportional to time-state-prices," *Economics Letters* (1981, Vol.8, No.1), pp.1–6.

Debreu, G. (1959), *Theory of value* (Wiley, 1959).

Demski, J.S. (1980), *Information analysis* (Addison-Wesley, 2. ed, 1980).

Fairley, W. and H.D. Jacoby (1975), "Investment analysis using the probability distribution of the internal rate of return", *Management Science* (1975, Vol.21, No.12), pp.1428–37.

Fama, E.F. (1977), "Risk adjusted discount rates and capital budgeting under uncertainty",, *Journal of Financial Economics* (1977, Vol.5, No.1), pp.3–24.

Haley, C.W. and L.D. Schall (1979), *The theory of financial decisions* (McGraw-Hill, 2. ed., 1979).

Hertz, D.B. (1964), "Risk analysis in capital investment", *Harvard Business Review*, (1964, Vol.42, No.1), pp.95–106.

——— (1968), "Investment policies that pay off", *Harvard Business Review* (1968, Vol.46, No.1), pp.96–108.

Hillier, F.S. (1963), "The derivation of probabilistic information for the evaluation of risky investments", *Management Science* (1963, Vol.9, No.3), pp.443–57.

——— (1969), *The evaluation of risky interrelated investments* (North-Holland, 1969).

Hirshleifer, J. (1970), *Investment, interest, and capital* (Prentice-Hall, 1970).
Keely, R.H. and R. Westerfield (1972), "A problem in probability distribution techniques for capital budgeting", *Journal of Finance* (1972, Vol.27, No.3), pp.703–09.
Leland, H.E. (1980), "On the use of risk adjusted discount rates", Working Paper (University of California, Berkeley, 1980).
Lewellen, W.G. and M.S. Long, (1972), "Simulation versus single-value estimates in capital expenditure analysis", *Decision Sciences* (1972, Vol.3, No.1), pp.19–33.
Lindsay, R. and A.W. Sametz (1963), *Financial management. An analytical approach* (R.D. Irwin, 1963).
Lutz, F. and V. (1951), *The theory of investment of the firm* (Princeton University Press, 1951).
Myers, S.C. (1968), "A time-state-preference model of security valuation", *Journal of Financial and Quantitative Analysis* (1968, Vol.3, No.1) pp.1–33.
Pratt, J.W. and J.S. Hammond III (1979), "Evaluating and comparing projects: simple detection of false alarms", *Journal of Finance* (1979, Vol.34, No.5), pp.1231–42.
Robichek, A.A. and S.C. Myers (1965), *Optimal financing decisions* (Prentice-Hall, 1965).
______ ______ (1966), "Conceptual problems in the use of risk adjusted discount rates", *Journal of Finance* (1966, Vol.21, No.4), pp.727–30.
Rubinstein, M.E. (1976), "The valuation of uncertain income streams and the pricing of options", *Bell Journal of Economics* (1976, Vol.7, No.2), pp.407–25.
Schall, L.D., G.L. Sundem and W.R. Geijsbeek Jr. (1978), "Survey and analysis of capital budgeting methods", *Journal of Finance* (1978, Vol.33, No.1), pp.281–87.
Sundem, G.L. (1974), "Analysis of simplifications in capital budgeting models", Working Paper (University of Washington, 1974).
Van Horne, J.C. (1966), "Capital budgeting decisions involving combinations of risky investments", *Management Science* (1966, Vol.13, No.2), pp.B84–92.
______ (1969), "The analysis of uncertainty resolution in capital budgeting for new products", *Management Science* (1969, Vol.15, No.8), pp.B376–86.